



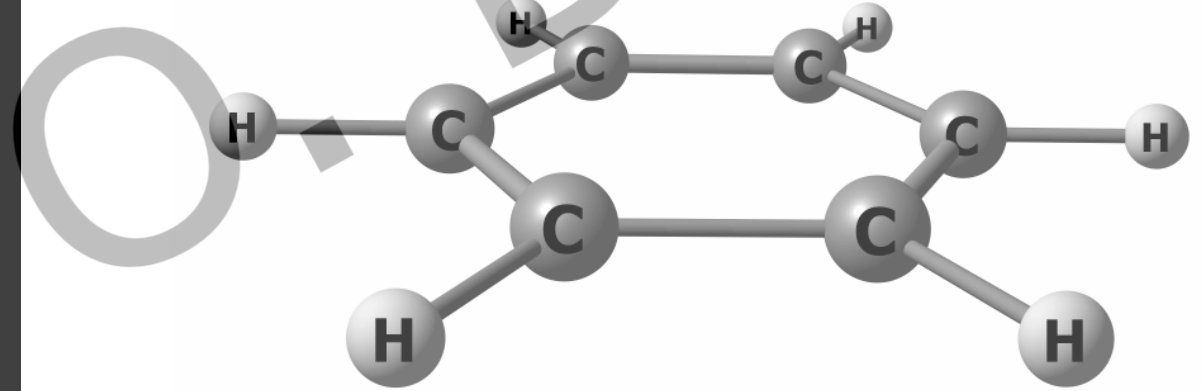
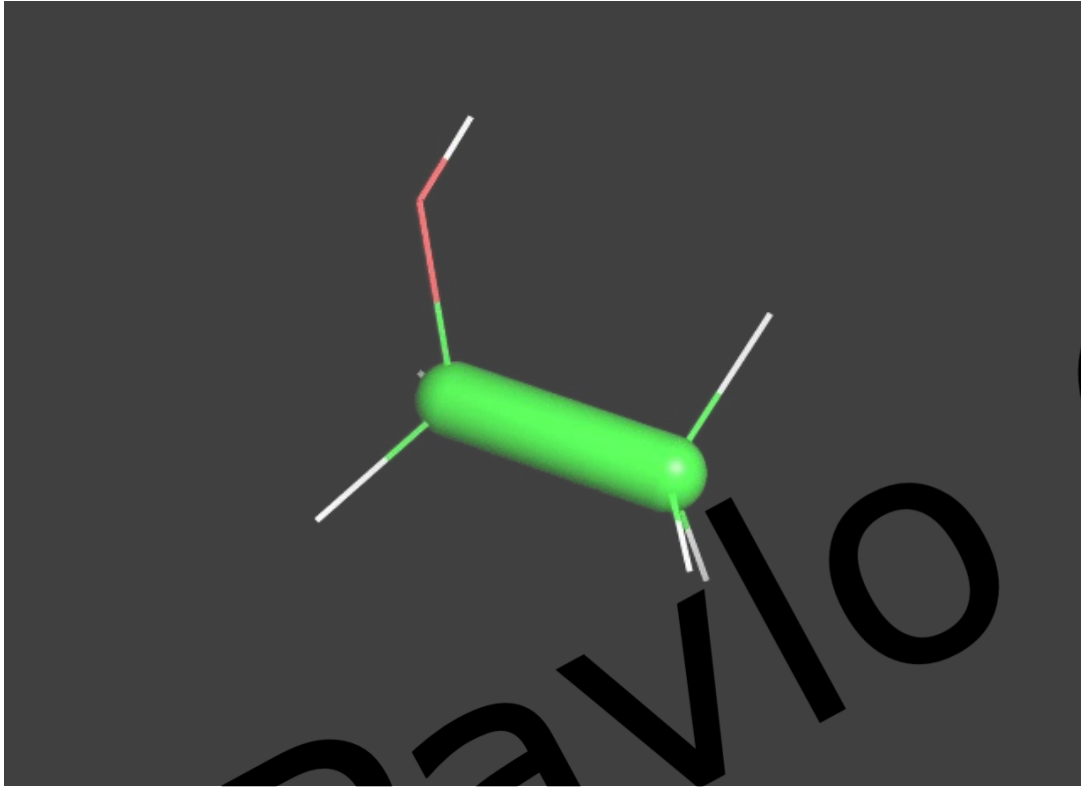
(Quasi-classical)  
molecular dynamics

**Pavlo O. Dral**  
Xiamen University, P.R. China

Visiting Professor in  
Nicolaus Copernicus University, Poland

5 July 2024

# Molecules are always in motion



To propagate molecular dynamics:

- We just need forces
- And solve iteratively Newton's equations of motion

Velocity Verlet algorithm:

$$F_{A,d} = M_A a_{A,d}$$

$$a_{A,d} = F_{A,d} \frac{1}{M_A}$$

$$\frac{\partial^2 x_{A,d}}{\partial^2 t} = F_{A,d} \frac{1}{M_A}$$

$$\mathbf{F} = -\nabla E(\mathbf{R}) \quad F_{A,d} = -\frac{\partial E}{\partial x_{A,d}}$$

$$\mathbf{x}(t + \Delta t) = \mathbf{x}(t) + \mathbf{v}(t)\Delta t + \frac{1}{2} \mathbf{a}(t)\Delta t^2$$

$$\mathbf{v}(t + \Delta t) = \mathbf{v}(t) + \frac{1}{2} [\mathbf{a}(t) + \mathbf{a}(t + \Delta t)]\Delta t,$$

where  $\mathbf{x}$  is the coordinate,  $\mathbf{v}$  – the velocity,  $\mathbf{a}$  – the acceleration,  $t$  – the time and

## Simplified schematic of the molecular dynamics algorithm

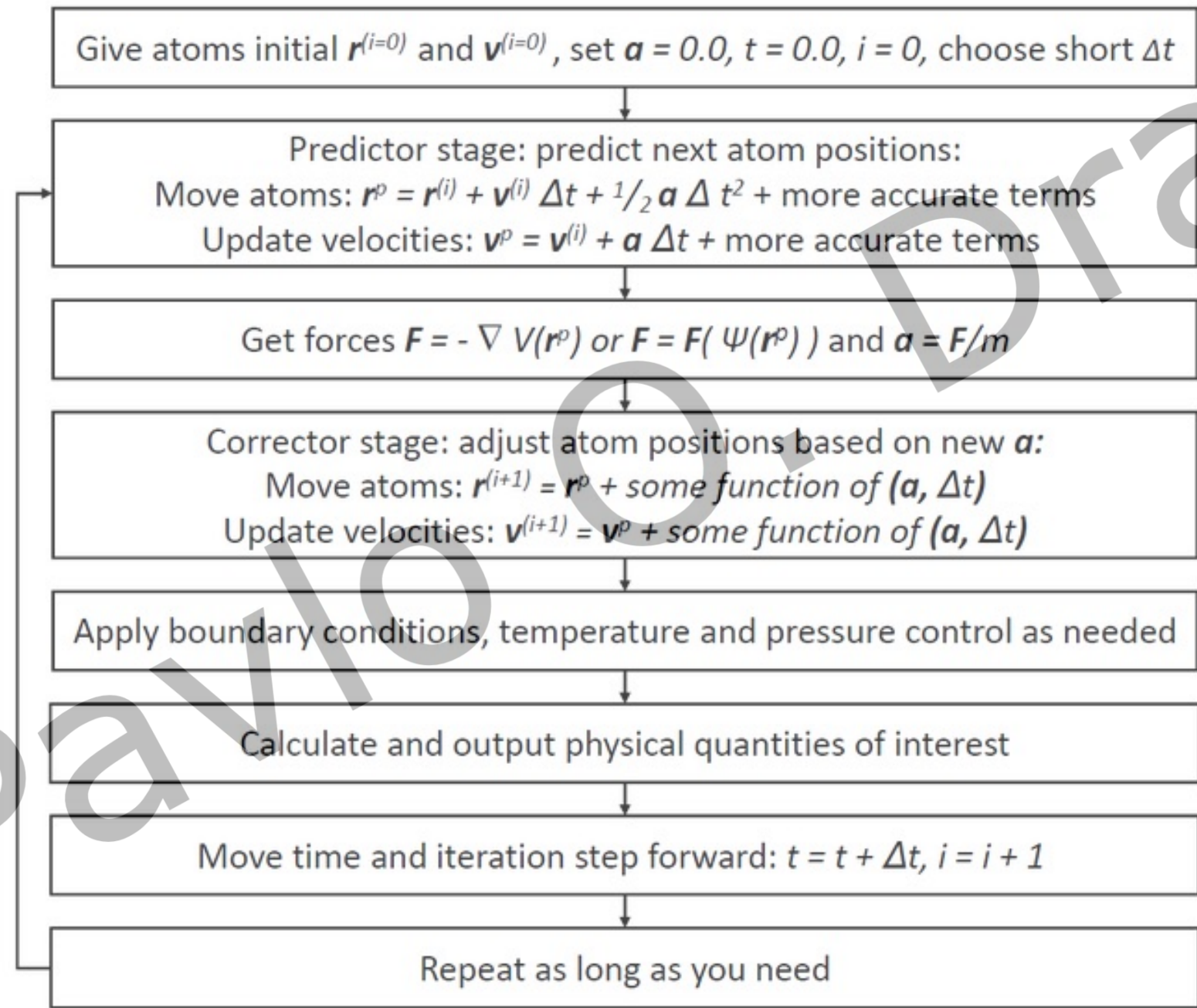


Figure credit:  
Knordlun, CC BY-  
SA 3.0, via  
Wikimedia  
Commons



## ① Example MD-1.

Propagate an MD trajectory of H<sub>2</sub> molecule with B3LYP/6-31G\*.

1. Visualize the trajectory. What is the range the bond length changes (min, max)?
2. How much time did it take to complete the calculations?

The MLatom@XACS input file:

```
MD
B3LYP/6-31G*
dt=0.5 # fs - time step
trun=500 # fs - propagation duration
initConditions=random
initXYZ='2
H 0 0 0
H 0 0 0.8'
```

How can we accelerate MD with ML?

Pavlo O. Dral

To propagate molecular dynamics:

- We just need forces →  
**can be obtained efficiently from machine learning potentials**

Velocity Verlet algorithm:

$$F_{A,d} = M_A a_{A,d}$$

$$a_{A,d} = F_{A,d} \frac{1}{M_A}$$

$$\frac{\partial^2 x_{A,d}}{\partial^2 t} = F_{A,d} \frac{1}{M_A}$$

$$\mathbf{x}(t + \Delta t) = \mathbf{x}(t) + \mathbf{v}(t)\Delta t + \frac{1}{2} \mathbf{a}(t)\Delta t^2$$

$$\mathbf{v}(t + \Delta t) = \mathbf{v}(t) + \frac{1}{2} [\mathbf{a}(t) + \mathbf{a}(t + \Delta t)]\Delta t,$$

where  $\mathbf{x}$  is the coordinate,  $\mathbf{v}$  – the velocity,  $\mathbf{a}$  – the acceleration,  $t$  – the time and

$$\mathbf{F} = -\nabla E(\mathbf{R}) \quad F_{A,d} = -\frac{\partial E}{\partial x_{A,d}} = -\frac{\partial E \text{ machine learning potential}(\mathbf{x})}{\partial x_{A,d}}$$

## Example MD-2.

Propagate an MD trajectory of H<sub>2</sub> molecule with ANI-1ccx.

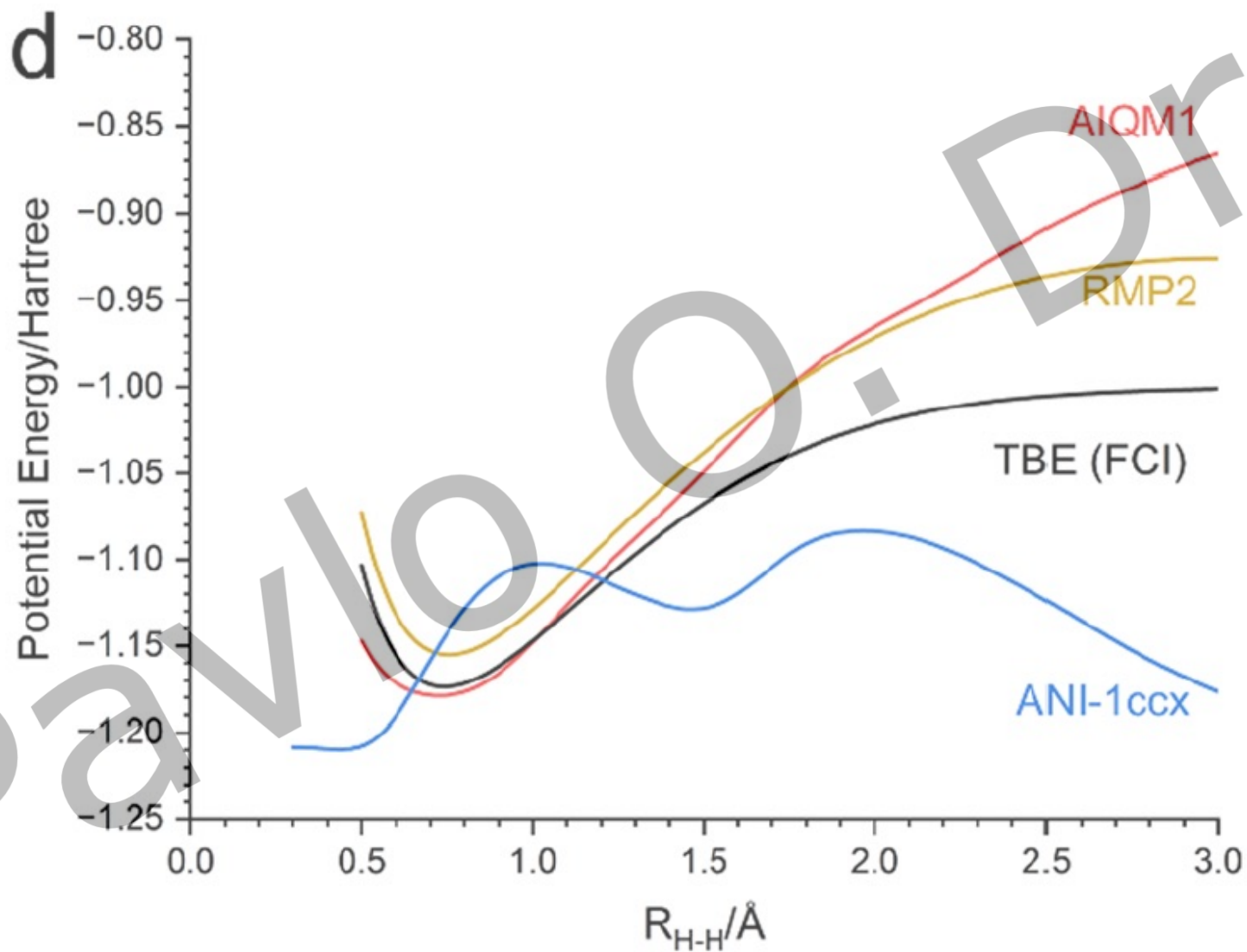
1. Visualize the trajectory. What is the range the bond length changes (min, max)?
2. How much time did it take to complete the calculations?
3. Compare time to DFT calculations.
4. Does ANI-1ccx produce physically-meaningful trajectory?

```
MD
ANI-1ccx
dt=0.5 # fs - time step
trun=500 # fs - propagation duration
initConditions=random
initXYZ='2
H 0 0 0
H 0 0 0.8'
```



Note the speed up for the hydrogen molecule is not so dramatic, but try something larger!

Pavlo O. Dral



## Task 5.4

Propagate an MD trajectory of H<sub>2</sub> molecule with your own ANI model trained on FCI data.

1. Visualize the trajectory. What is the range the bond length changes (min, max)?
2. How much time did it take to complete the calculations?
3. Does this ANI model produce physically-meaningful trajectory?

Modify the input file from [previous task](#) as needed.

MLatom implements Nosé–Hoover chain, where the velocities of particles are controlled by a chain of additional degrees of freedom to guarantee the canonical sampling of the original system. The equations of motion are shown below:

$$\mathbf{v}_i = \frac{\mathbf{p}_i}{m_i}, \quad \dot{p}_{\xi_1} = \left( \sum_i \frac{\mathbf{p}_i^2}{m_i} - N_f kT \right) - p_{\xi_1} \frac{p_{\xi_2}}{Q_2},$$

$$\dot{\mathbf{p}}_i = \mathbf{F}_i - \mathbf{p}_i \frac{p_{\xi_1}}{Q_1}, \quad \dot{p}_{\xi_k} = \left( \frac{p_{\xi_{k-1}}^2}{Q_{k-1}} - kT \right) - p_{\xi_k} \frac{p_{\xi_{k+1}}}{Q_{k+1}},$$

$$\xi_i = \frac{p_{\xi_i}}{Q_i},$$

$$\dot{p}_{\xi_M} = \left( \frac{p_{\xi_{M-1}}^2}{Q_{M-1}} - kT \right).$$

## ① Example MD-4.

Propagate an MD trajectory of H<sub>2</sub> molecule with [your own ANI model trained on FCI data](#) at 300 K using the NVT ensemble.

1. Plot how the total energy changes compared to the [simulation in the NVE ensemble](#). What is the major difference?
2. Plot how the temperature changes too.
3. Is it expected?

You would need to add the following lines to your input file from [previous task](#):

```
temperature=300  
ensemble=NVT  
thermostat=Nose-Hoover
```

## ① Note

You can find total energy in the files `traj.etot` and temperature in the files `traj.temp`.



**Missing zero-point vibrational energy**

Pavlo O. Dral

## ① Example MD-5.

Start MD of H<sub>2</sub> molecule with [your own ANI model trained on FCI data](#) from the optimized geometry (with the same model, you can take it from your [previous exercise](#)) using the **NVE ensemble**, and using the initial temperature of 0 K.

What happens to MD?

Pavlo O. Dral

Pavlo O. Dral

**Quasi-classical MD recovers ZPVE**

 The picture can't be displayed.

MD17 – MD at 500 K  
WS22 – Wigner sampling

$$P_w(\mathbf{Q}, \mathbf{P}) = \prod_{i=1}^{N_F} \frac{1}{\pi \hbar} \exp\left(-\frac{1}{\hbar \omega_i} (\omega_i^2 Q_i^2 + P_i^2)\right)$$

$$Q_i = \mu_i^{1/2} q_i, P_i = \mu_i^{-1/2} p_i$$

## Example MD-6.

Run quasi-classical trajectory MD of H<sub>2</sub> molecule with [your own ANI model trained on FCI data](#) from the optimized geometry (with the same model, you can take it from your [previous exercise](#)).

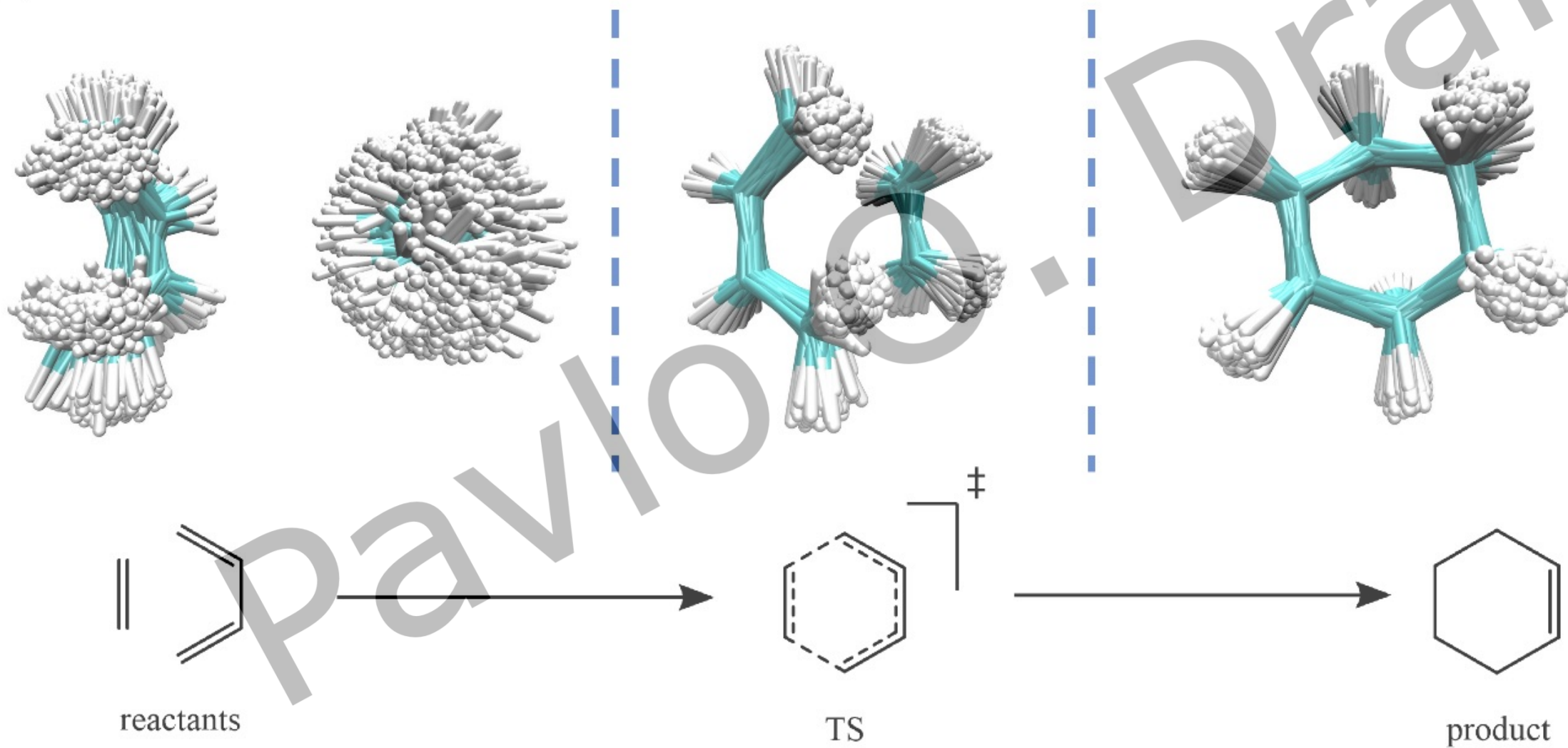
For this, you first need to do freq calculations on the optimized geometry. These calculations will produce the `freq1.json` file which you can use in the following input file:

```
MD
MLmodelIn=energies_ani.pt
MLmodelType=ANI
dt=0.5 # fs - time step
trun=100 # fs - propagation duration
initTemperature=0
initConditions=Wigner
normalModeFile=freq1.json
```

How is this MD different to the previous one? What happens to MD?



(a)



**Nuclear quantum effects are not taken into account in MD!**

**Methods like PIMD are required**

Pavlo O. Dral

# Molecular Dynamics with Constrained Nuclear Electronic Orbital Density Functional Theory: Accurate Vibrational Spectra from Efficient Incorporation of Nuclear Quantum Effects



Xi Xu, Zehua Chen, and Yang Yang\*

Cite This: *J. Am. Chem. Soc.* 2022, 144, 4039–4046

Read Online

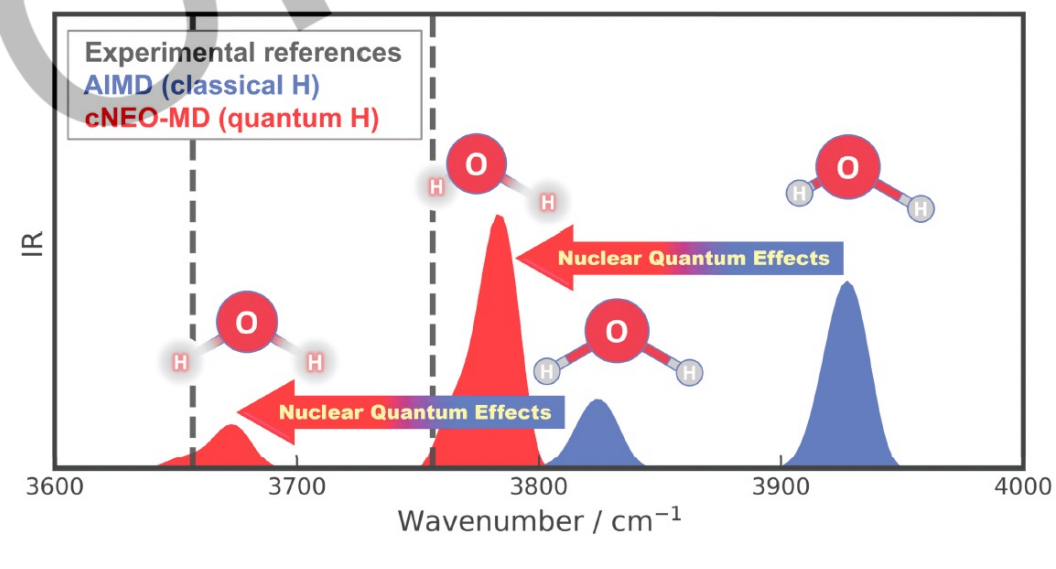
ACCESS |

Metrics & More

Article Recommendations

Supporting Information

**ABSTRACT:** Nuclear quantum effects play a crucial role in many chemical and biological systems involving hydrogen atoms yet are difficult to include in practical molecular simulations. In this paper, we combine our recently developed methods of constrained nuclear–electronic orbital density functional theory (cNEO-DFT) and constrained minimized energy surface molecular dynamics (CMES-MD) to create a new method for accurately and efficiently describing nuclear quantum effects in molecular simulations. By use of this new method, dubbed cNEO-MD, the vibrational spectra of a set of small molecules are calculated and compared with those from conventional *ab initio* molecular dynamics (AIMD) as well as from experiments. With the same formal scaling, cNEO-MD greatly outperforms AIMD in describing the vibrational modes with significant hydrogen motion characters, demonstrating the promise of cNEO-MD for simulating chemical and biological systems with significant nuclear quantum effects.

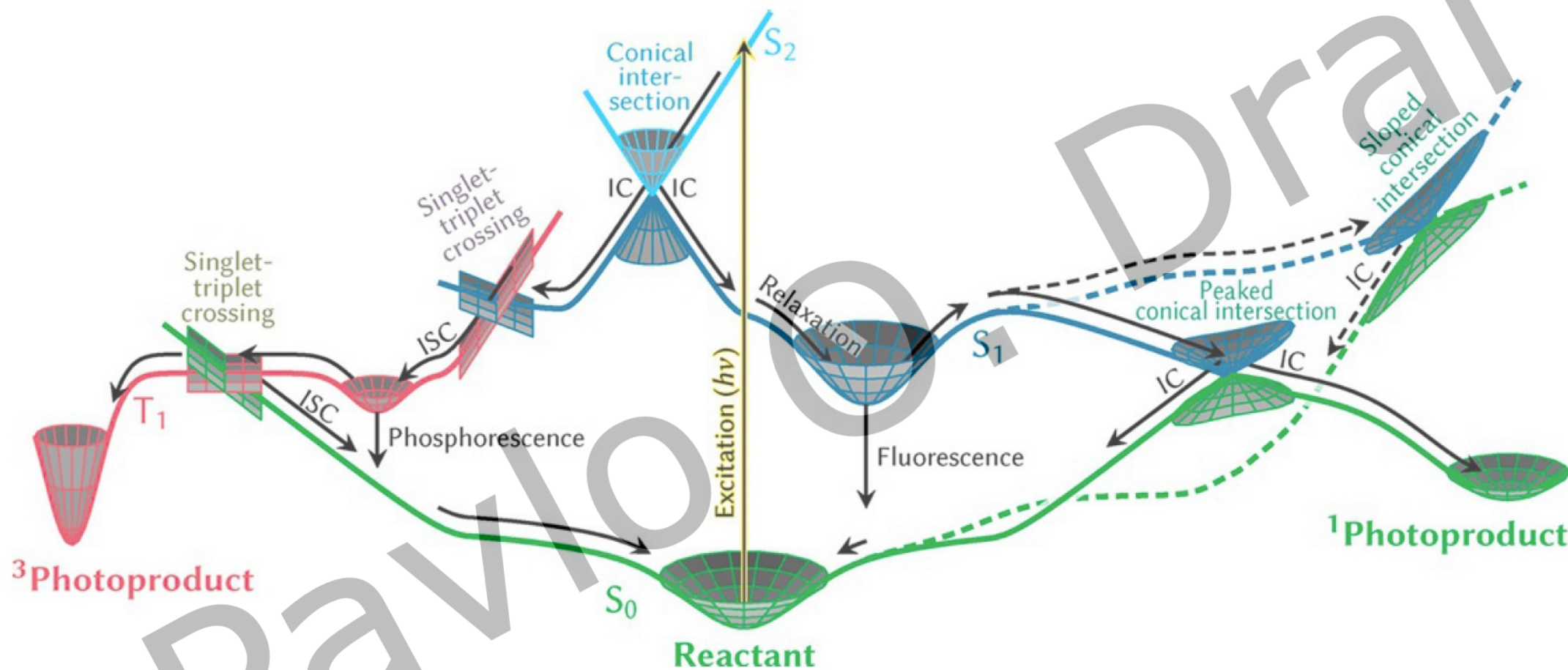


the promise of cNEO-MD for simulating chemical and biological

**Transitions between electronic states also often have to be taken into account, particularly, when simulating photophysical and photochemical processes**

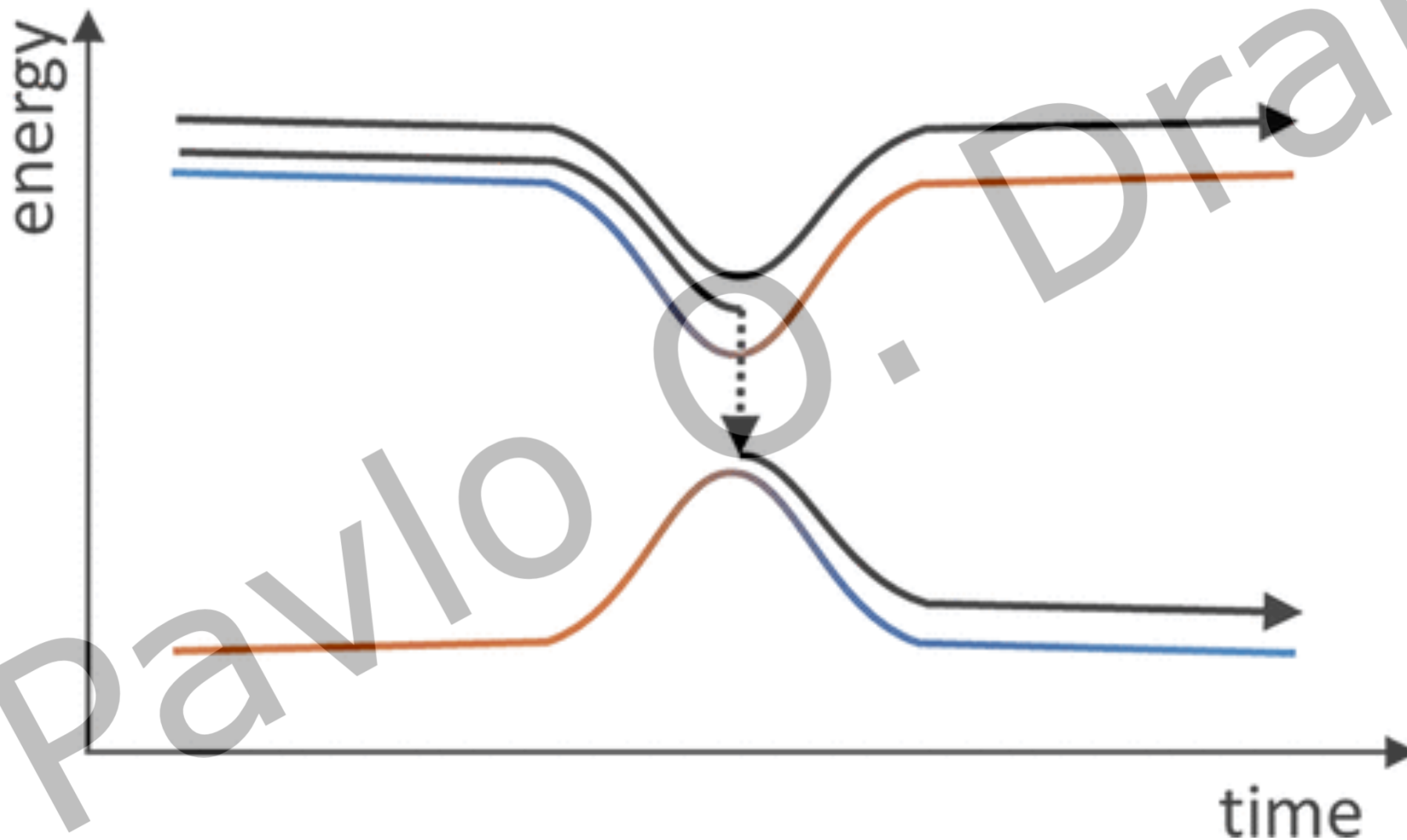
Pavlo O. Dral





**FIG. 3** A typical potential energy surface in NAMD. Reproduced from S. Mai, L. González, *Angew. Chem. Int. Ed.* 59 (2020) 16832–16846, under CC BY 4.0.



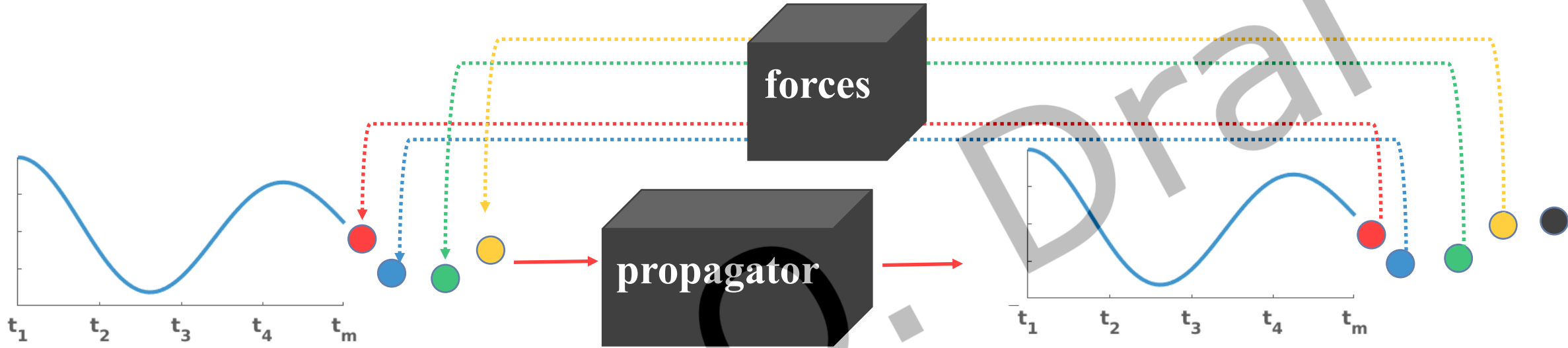


J. C. Tully, *J. Chem. Phys.* **1990**, 93, 1061

R. Crespo-Otero and M. Barbatti, *Chem. Rev.* **2018**, 118, 7026

How can we accelerate MD with ML?

Pavlo O. Dral

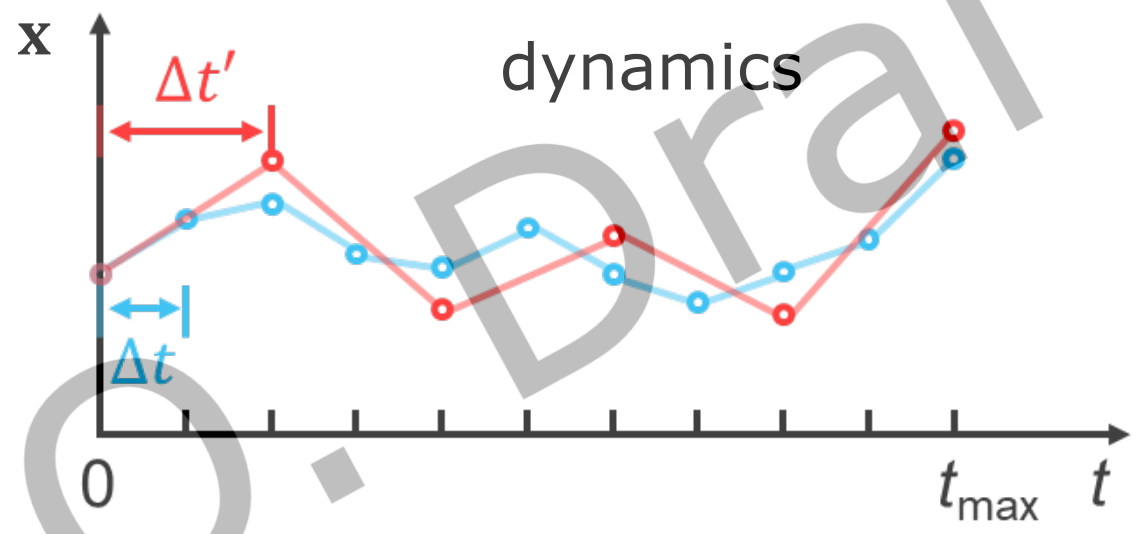


$$F_{A,d} = - \frac{\partial E}{\partial x_{A,d}} = - \frac{\partial E_{\text{machine learning potential}}(\mathbf{x})}{\partial x_{A,d}}$$

# Can we do better?

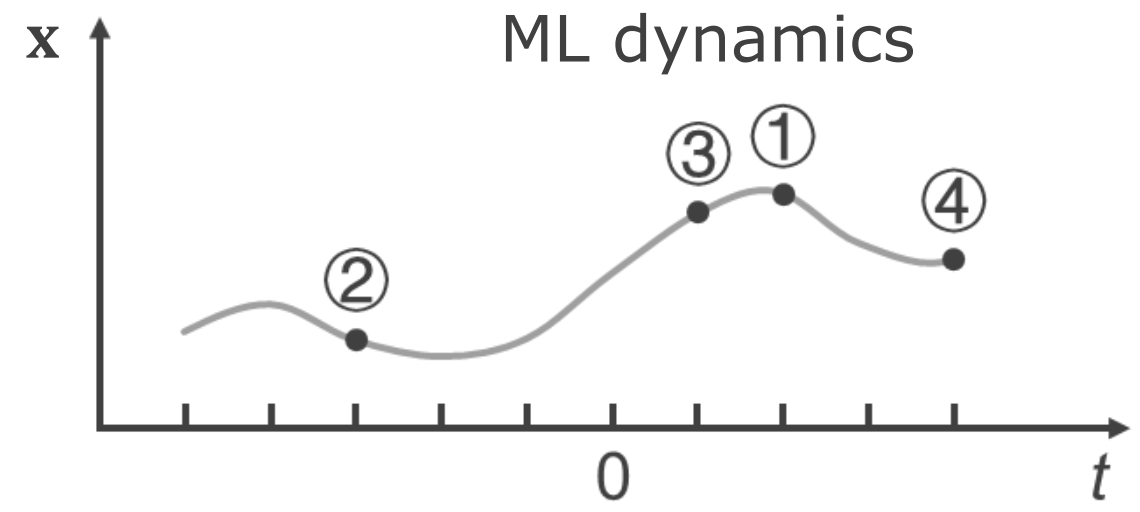
dynamics propagation is

- Iterative (non-parallelizable)
- Depends on time step
- Discrete



Directly learning dynamics

$$\mathbf{x}_t = f(\mathbf{x}_0, \mathbf{v}_0, t)$$



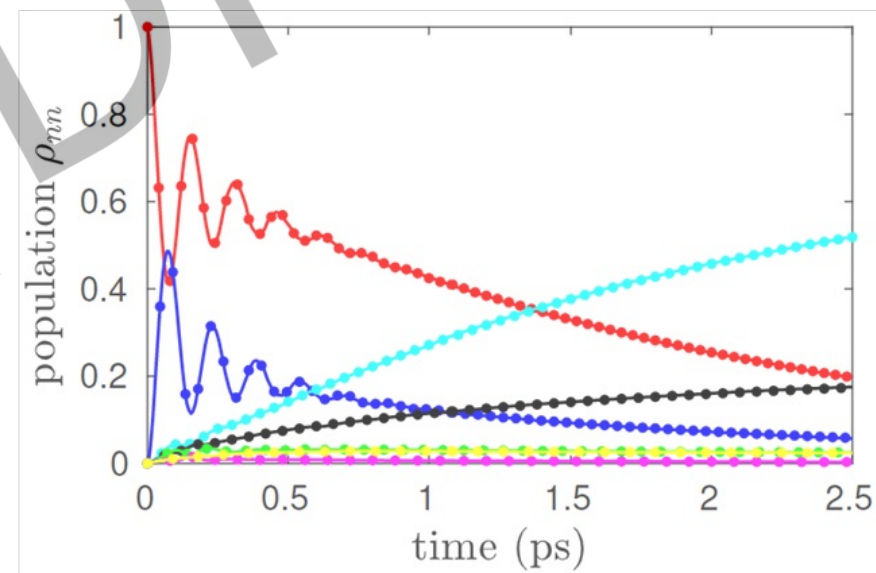
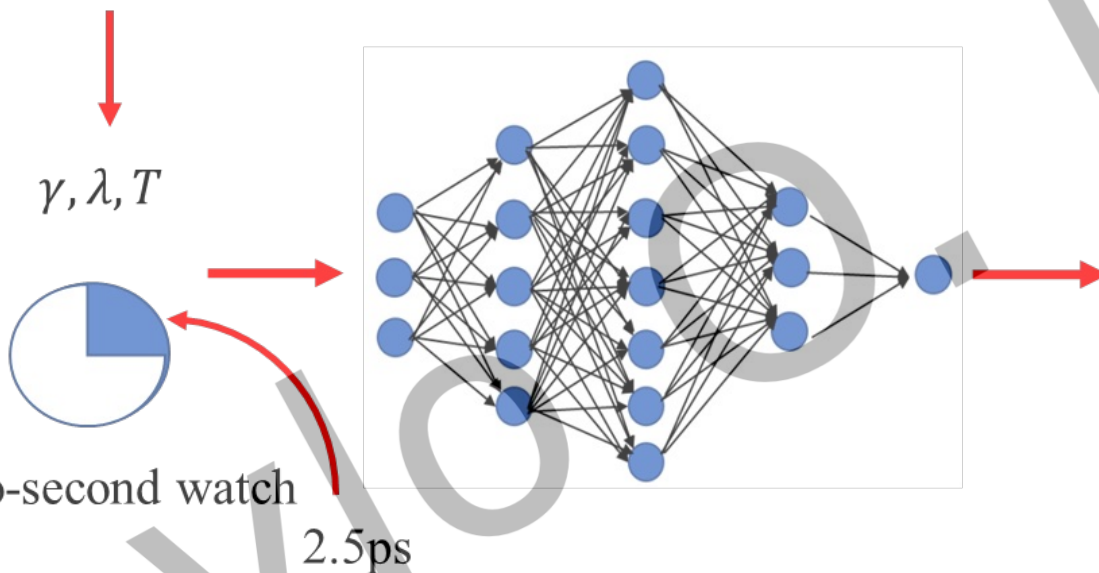
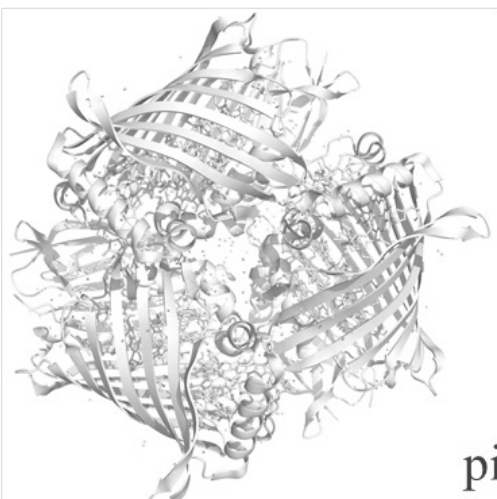
$\gamma$  = characteristic frequency

$\lambda$  = reorganization energy

$T$  = temperature

$$\rho(\text{time}) = f[\text{time}; \text{simulation parameters}]$$

PDB code: 3ENI

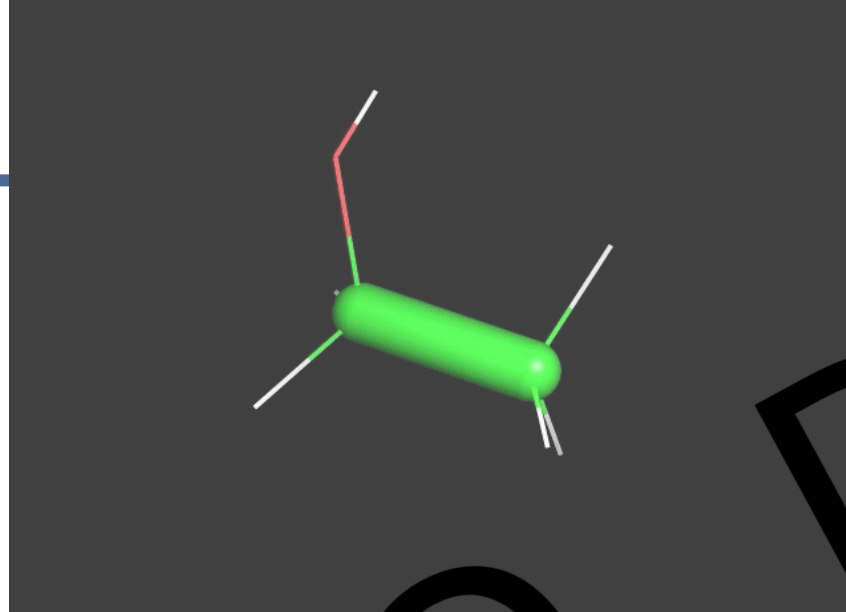


**Dots: Reference**  
**Line: AI-QD**

## 7-sites Fenna–Matthews–Olson (FMO) complex

A. Ullah, P. O. Dral. Predicting the future of excitation energy transfer in light-harvesting complex with artificial intelligence-based quantum dynamics. *Nat. Commun.* **2022**, *13*, 1930

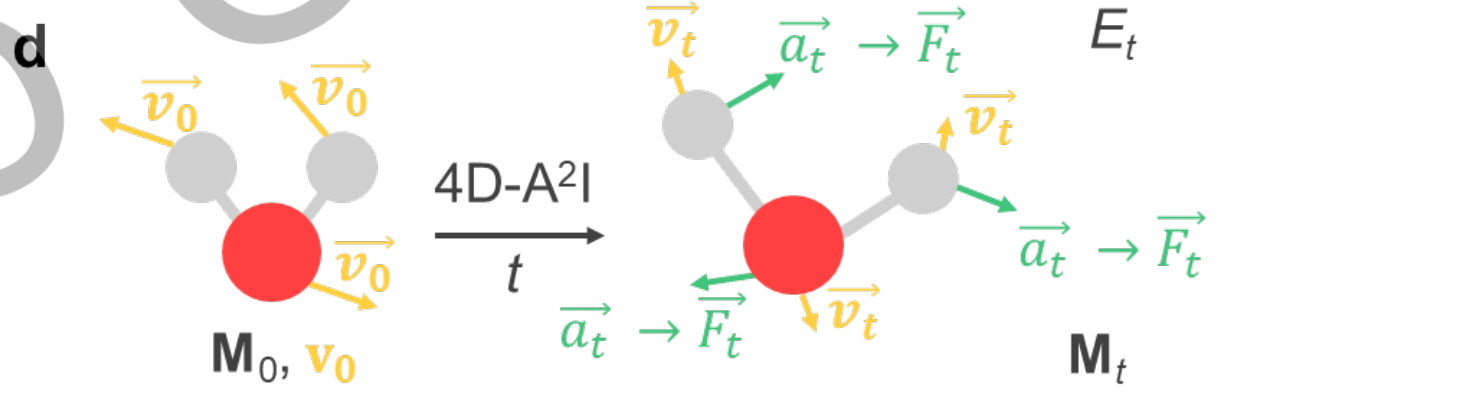
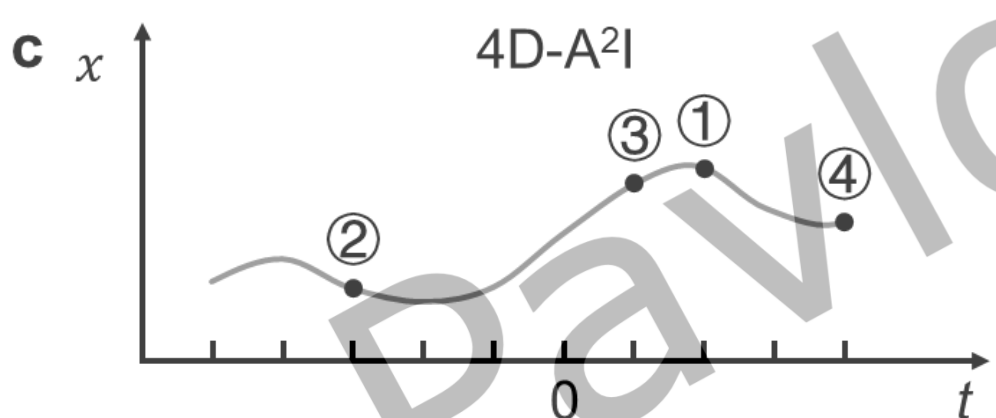
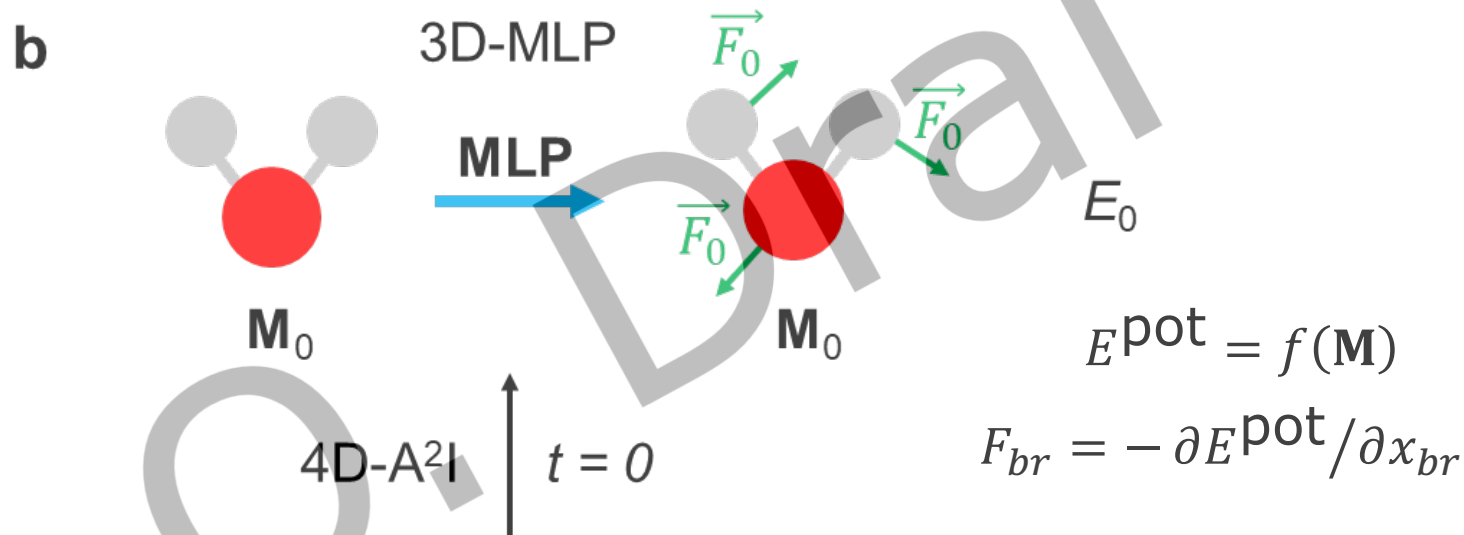
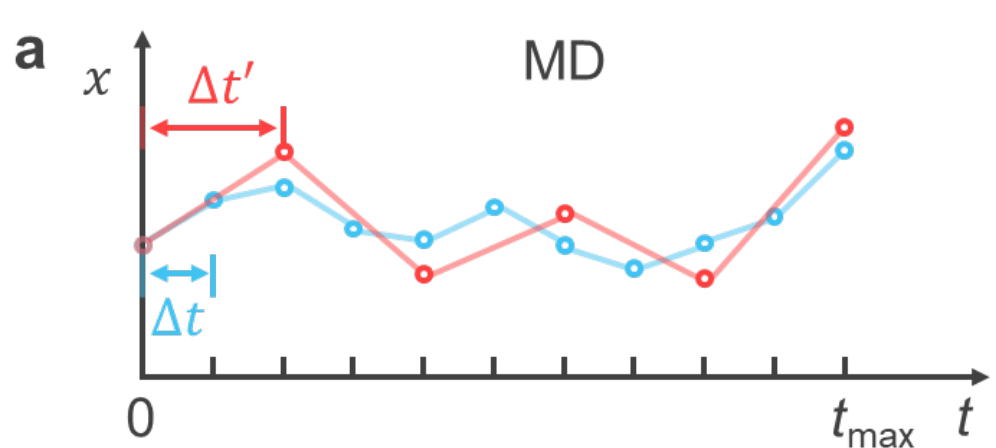




# ***Direct learning of molecular dynamics***

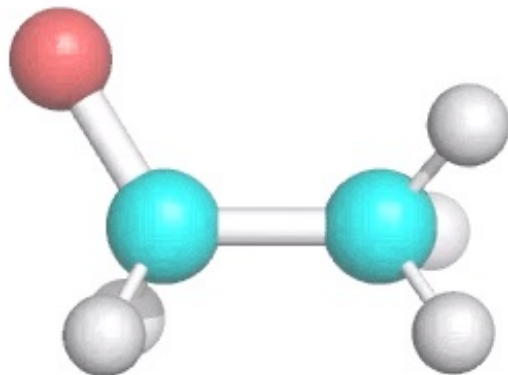
***or***

# ***Meet 4D spacetime atomistic artificial intelligence (4D-A<sup>2</sup>I) models***



$$\mathbf{x}_t = f(\mathbf{x}_0, \mathbf{v}_0, t) \longrightarrow v_{br,t} = \frac{dx_{br,t}}{dt} \longrightarrow a_{br,t} = \frac{dv_{br,t}}{dt} \longrightarrow F_{br,t} = m_b a_{br,t}$$

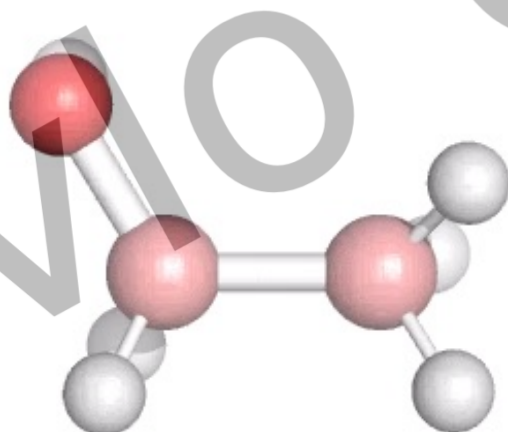
3D MD



Very slow

## The direct learning of molecular dynamics with 4D-spacetime GICnet models

4D-A<sup>2</sup>I

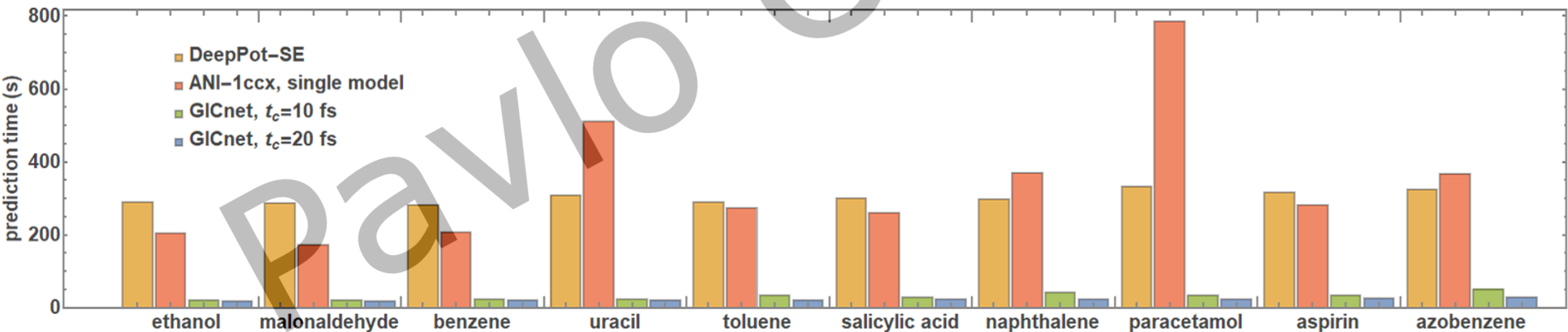
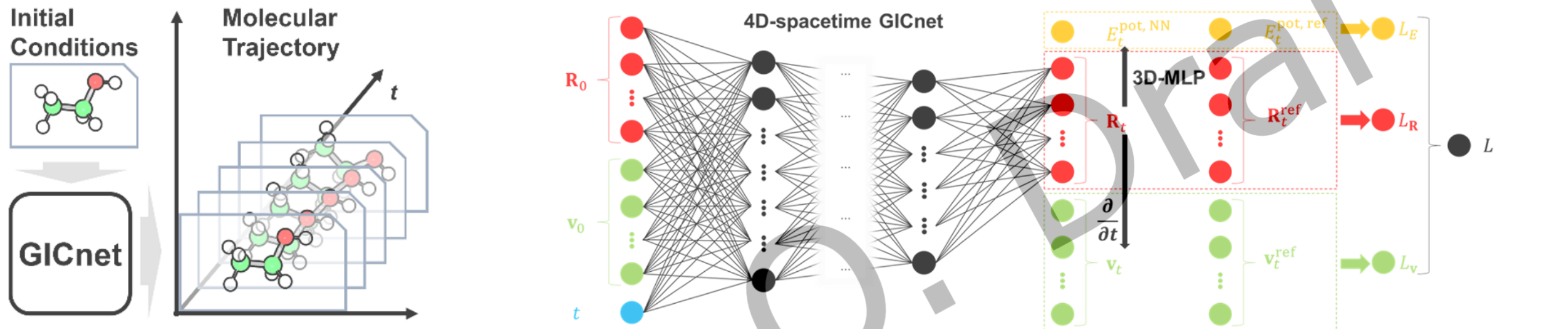


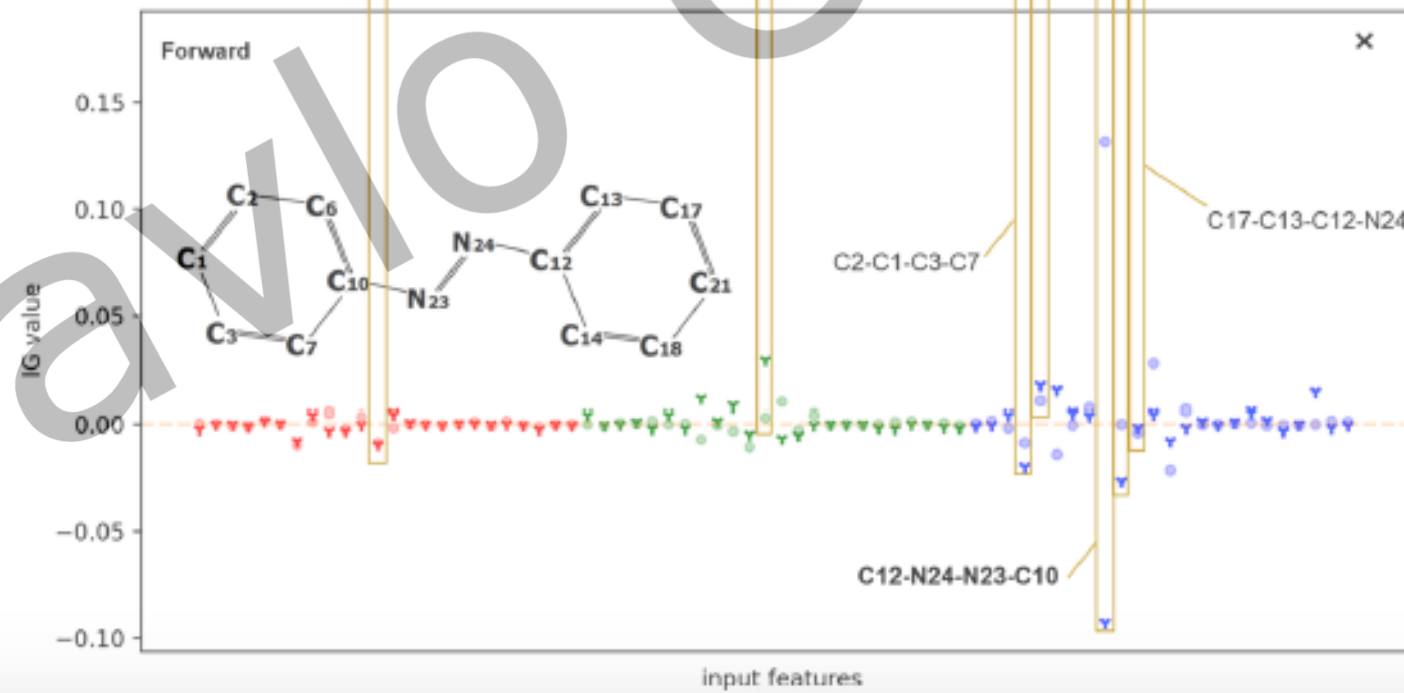
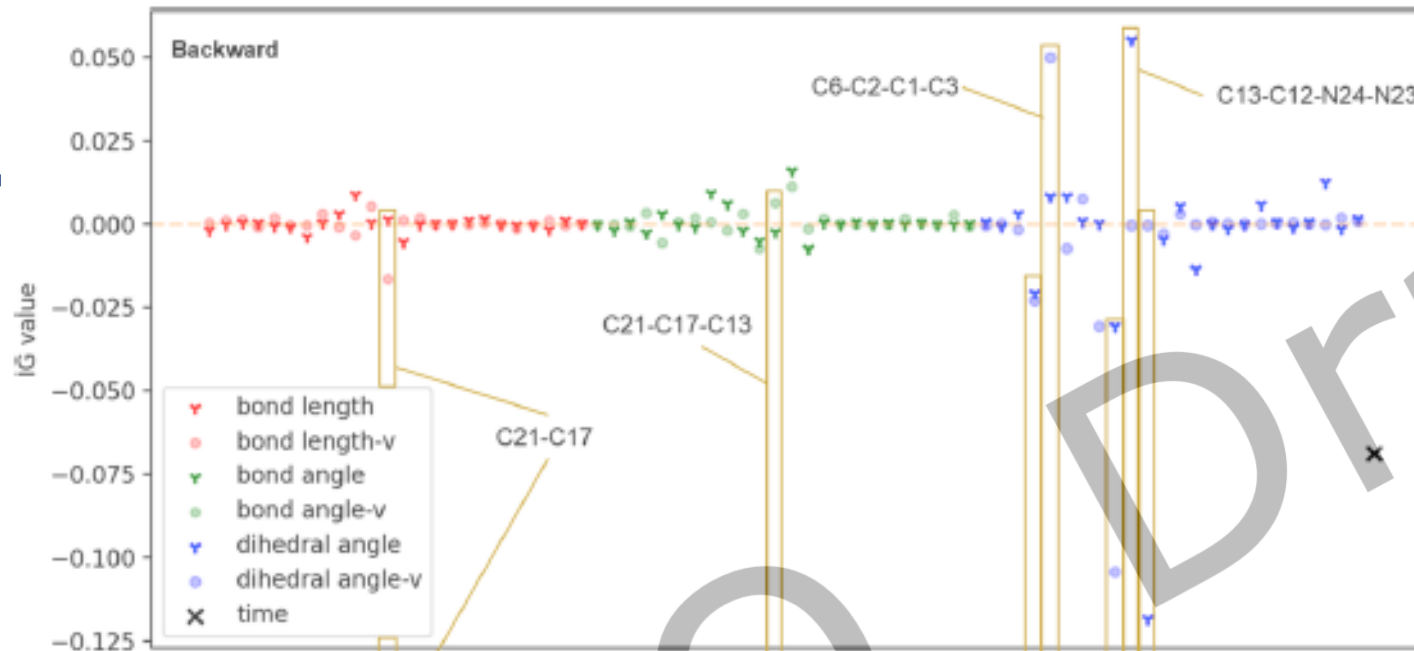
0.05 fs

Very fast, e.g., 1 ps trajectory (time step 0.05 fs) within 1 minute



Fuchun Ge





# Can we do even better?

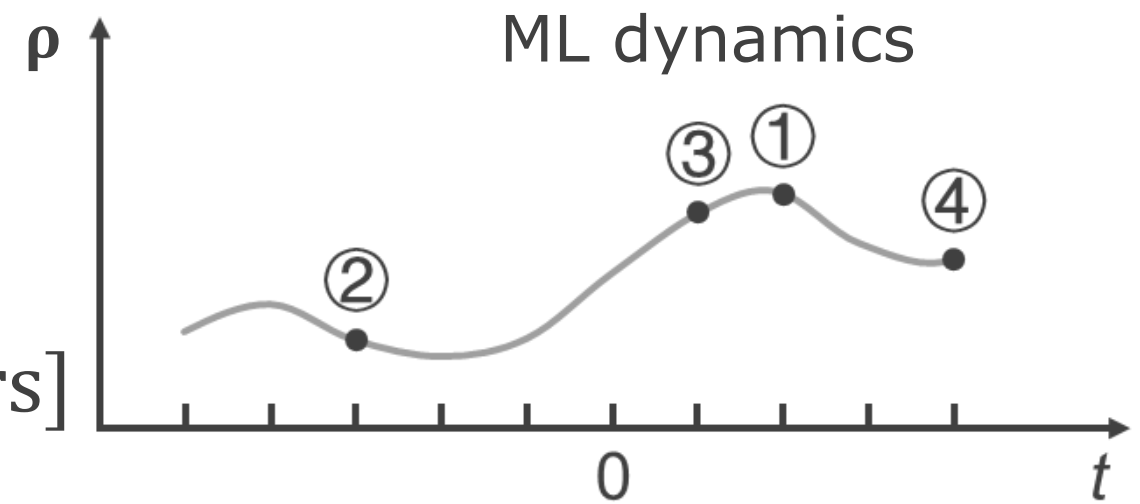
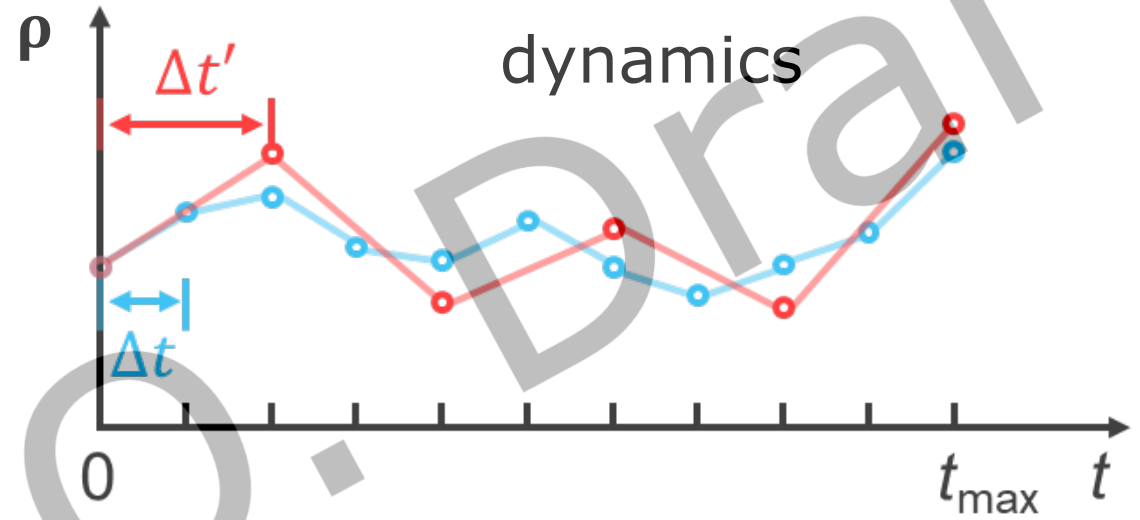
~~$$\rho(t) = f[\rho(t - \Delta t)]$$~~

dynamics propagation is

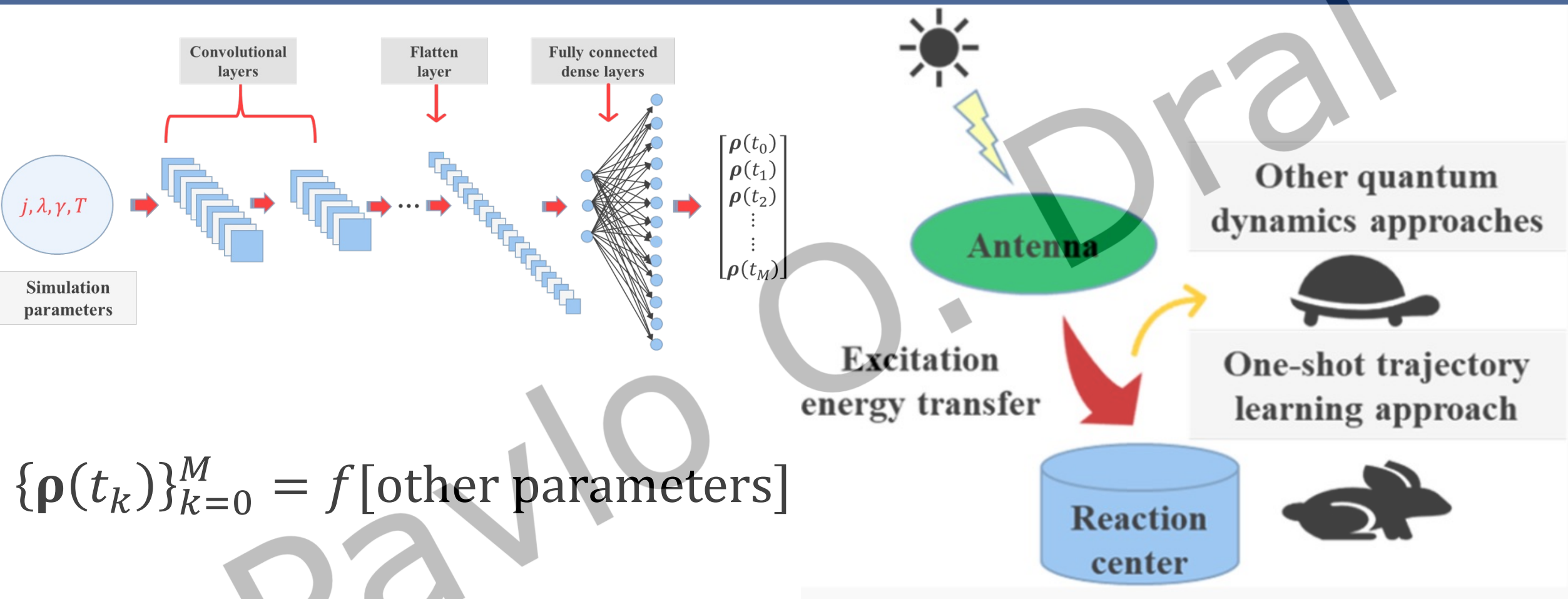
- ~~computationally expensive~~
- ~~recursive (iterative)~~

~~$$\rho(t) = f[t; \text{other parameters}]$$~~

$$\{\rho(t_k)\}_{k=0}^M = f[\text{other parameters}]$$







- **10 ps long dynamics in just 70 ms**
- good for massive simulation in parameter space